1 **Medieval Mathematics: An Overview**

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2 **Objectives**

For the medieval period (600-1600 AD):
- Understand the numerals and notations that were used for mathematics worldwide
- Understand the mathematical technology that was available
- Track changes that allowed these to evolve into the system we have today

3 **Prologue: Math Is A Technology**
- A culture's mathematics is interdependent with its other technologies
- Mathematics can affect technological development
- Technology can affect mathematical development

4 **Prologue: Numerals, Numbers, And Digits**
- Number: a cardinal quantity
- Numeral: a symbol that represents a particular cardinal quantity
- Digit: a symbol used as part of a numeral
- Decimal digit: our familiar 0-9 digits

5 **Prologue: Positional Numeration Systems**
- If a digit can represent a different number based on its position, the system is positional
- E.g. the 2 in 2506 represents 2000, while the 2 in 4025 represents 20
- Not all numeration systems are positional!

6 **Prologue: The Role Of Zero**
- When the numeral zero is used in a positional numeration system to indicate an empty position, it is a placeholder zero
- When the numeral zero is able to be used in computations (e.g. $0 \times 5$), it is a computational zero

7 **Prologue: The System We Have Now**
- Base 10, positional (place-value system)
- Placeholder and computational 0
- 10 distinct digits that are abstract symbols and do not represent the number visually
- Algebra: symbolic notation using letters for unknowns and + - × ÷ = for operations and equality, fractions in any denomination with fraction bar, roots of any index
- Most of this did not exist in 600 AD!

8 **Snapshot: 600 AD**

1 Numeration

2
- Europe: Roman numerals, finger numerals
- Asia: Chinese stick numerals, glyph numerals
- India: precursors of our numerals
- Hebrew and Greek numerology
- Mesoamerica: Mayan numerals

3 Computation

4
- Europe: line abacus/counting board, various abaci

- Asia: grid abacus,
  suan-pan/soroban/supan (bead abacus)
- Mesoamerica: Nepōhualtzintzin

9 **Snapshot: 1600 AD**

1 Numeration
2 – Hindu-Arabic numerals (our modern ones) are standardized and prevalent
– Numerology still in use
3 Computation
4 – Symbolic computation is commonplace
– Abaci of various types are still in use
– Algebraic notation is becoming popular

10 **Numerals and Numeration Systems**

11 **Roman Numerals**

- Base 10, not positional, no zero
- Additive and subtractive system
- Highest numeral is 1,000,000 through
  extended system

12 **Hebrew and Greek Letter Numerals**

- Base 10, non-positional, no zero
- Letters represent numerals of varying denominations
- Combinations of letters create numerals additively
- Correspondence between numerals and words enables numerology

13 **Finger Numerals**

- Non-positional; no zero
- Used for business negotiations
- Hidden under sleeves; read by touch
- Ancient dirty math joke

14 **Chinese Stick Numerals**

- Base 10, positional numeration
- Computational and placeholder zero
- Decimal fractions
- Used on grid abacus
- Operations carried out via manipulation of sticks inside grid squares

15 **Mayan Numerals**

- Positional base 20 system with sub-base of 5
- Placeholder zero
- Alternate system not pure base 20; 2$^{nd}$ column base 18 to enable astronomical calculations

16 **Hindu-Arabic Numerals**

- Positional with computational and placeholder zero
- Originally influenced by Chinese stick numerals
- Arrived in Europe via contact with Muslim civilizations in the Middle East and Spain

– Thought to be demonic, just as they are today ;)

17 **Computation and Algebra**

18 **Margarita Philosophica**

– This woodcut from the Margarita Philosophica of Gregor Reisch, 1503, depicts Boethius and Pythagoras in a computational competition, judged by a female figure symbolizing arithmetic.
– Pythagoras (right) is using a counting board
– Boethius (left) is using Hindu-Arabic numerals and fractions

19 **Counting boards**

20 **Using a counting board**

– Counting boards could be elaborate or simple.
– Lines represent powers of 10.
– The placement of a calculus on the board indicates its value. Placed on the line, it indicates the Roman numeral written on that line. Placed between two lines, it indicates the "5" value that's between those lines.

21 **Fractions**

– Until decimal fractions were invented, Egyptian fractions were used
– Egyptian fractions have numerators of 1 (except 2/3)
– Fractions that don't have a numerator of 1 are represented by a sum of fractions with numerator 1, e.g. $\frac{3}{4} = \frac{1}{2} + \frac{1}{4}$
– The Rhind Papyrus (1650 B.C.) has fraction tables
– Fibonacci wrote an algorithm for Egyptian fractions in 1202 A.D.

22 **Algebra**

– Systems of two linear equations in two variables, and Diophantine equations (single linear equations in two variables) have been solvable since antiquity
– The word "algebra" comes from Al-Khwarizmi's classic textbook *Al-Kitāb al-mukhtaṣar fī hīsāb al-ğabr wa'l-muqābala (The Compendious Book on Calculation by Completion and Balancing),* pictured at left

23 **Solving Equations**

– Quadratic equations have been solved since ancient Babylon (positive real solutions only)
– Brahmagupta (628 A.D.): quadratic formula (positive real solutions only), Simon Stevin (1594 A.D.) for all solutions
– Chinese could solve some cubic equations by 2nd century B.C. and general cubics and some quartics by 7th century A.D.
– Italian mathematicians Del Ferro, Ferrari, Tartaglia, and Cardano raced to solve the general cubic in the 1500's A.D.
– French mathematicians Viète and Descartes worked on a trigonometric solution to the general cubic

24 **Modern Algebraic Notation**

– Islamic Golden Age: fraction bar
– 14th century: plus and minus signs
– 1500's: "co" (cosa = "thing"), "ce" (censo), and "cu" (cubo) for $x, x^2, x^3$

- c. 1520: Christoph Rudolff, radical sign
- 1557: Robert Recorde, equals sign
- 1585: Simon Stevin, decimal notation
- 1591: François Viète, modern algebraic notation
- 1631: William Oughtred, multiplication sign

25 **Questions?**